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ASYMPTOTIC SOLUTION OF A CLASS OF INTEGRAL EQUATIONS AND ITS APPLICATION TO CONTACT PROBLEMS FOR CYLINDRICAL ELASTIC BODIES

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A special class of integral equations of the first kind with irregular difference kernel of complex structure dependent on a nondimensional parameter λ is considered. The asymptotic solution of this integral equation is constructed for large values of λ as a double series in powers of λ^{-1} and $\ln \lambda$.

The obtained results are utilized to study axisymmetric problems of the interaction between a stiff belt and the surface of an infinite elastic cylinder, as well as the interaction between a stiff bushing and the surface of an infinite cylindrical cavity in elastic space.

Finally, under the customary assumptions of Hertz theory, the problem of interaction between an elastic belt and infinite elastic cylinder is examined on the basis of the solution of the first two problems.

1. **Investigation of the structure of the solution of the integral equation and construction of the asymptotic solution for large values of the parameter λ.** Let us consider an integral Eq. of the form

$$\int_{-1}^{1}\left\{-\ln\frac{|x-t|}{\lambda}+a_{20}\frac{|x-t|}{\lambda}+a_{30}+F\left(\frac{x-t}{\lambda}\right)\right\}\varphi(t)\,dt=\pi f(x)\qquad(|x|\leqslant 1)\tag{1.1}$$

$$F(y)=\ln|y|\,F_1(y)+|y|\,F_2(y)+F_3(y)\tag{1.2}$$

The functions $F_i(y)$ will be continuous with all their derivatives for all values $-2/\lambda\leqslant y=(x-t)/\lambda\leqslant 2/\lambda$ and will behave as $O(y^2)$ for $y\to 0$.

Hence it follows that the function $F(y)\in H_1^{\alpha}(-1,1)$, $0<\alpha<1$ where $H_n^{\alpha}(-\beta,\beta)$ denotes the space of functions whose n-th derivative satisfies the Hölder condition with exponent α for $|x|\leqslant\beta$.

We shall moreover assume that $f(x)\in H_p^{\alpha}(-1,1)$, $\alpha>0$, $p\geqslant 1$.

Following [1], let us represent (1.1) as an equivalent integral equation of the second kind

$$\omega(x)=\frac{P}{\pi}-\frac{1}{\pi}\int_{-1}^{1}\frac{f'(t)\sqrt{1-t^2}}{t-x}\,dt+\frac{1}{\pi^2}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{t-x}\,dt\int_{-1}^{1}\Big\{a_{20}\,\mathrm{sgn}\,(t-y)+$$

$$+\lambda F_t'\left(\frac{t-y}{\lambda}\right)\Big\}\frac{\omega(y)}{\lambda\sqrt{1-y^2}}dy, \qquad \omega(x)=\varphi(x)\sqrt{1-x^2} \tag{1.3}$$

The quantity P is determined either from the condition of compliance with the solution of (1.1) found from (1.3), or equivalently, by means of the Formula [1]:

$$P=\frac{1}{\ln 2\lambda+a_{30}}\left\{\int_{-1}^{1}\frac{f(t)\,dt}{\sqrt{1-t^2}}-\frac{1}{\pi}\int_{-1}^{1}\frac{\omega(x)\,dx}{\sqrt{1-x^2}}\int_{-1}^{1}\left[a_{20}\frac{|t-x|}{\lambda}+F\left(\frac{t-x}{\lambda}\right)\right]\frac{dt}{\sqrt{1-t^2}}\right\} \tag{1.4}$$

wherein

$$P=\int_{-1}^{1}\frac{\omega(t)}{\sqrt{1-t^2}}dt \tag{1.5}$$

Now, let us prove that if the solution of (1.1) in the class L $(-1, 1)$ exists then the function $\omega(x)\in C$ $(-1, 1)$ holds for any value of $\lambda\in(0, \infty)$. To do this, it is evidently sufficient to prove that the integral operator in the right-hand side of (1.3) operates from the space C $(-1, 1)$ into C $(-1, 1)$.

It has been shown in [1] that the integral

$$J(x)=\int_{-1}^{1}\frac{\gamma(t)\sqrt{1-t^2}}{t-x}dt\in C_m(-1,1), \quad \text{если } \gamma(t)\in H_m^{\alpha}(-1,1), \quad \alpha>0 \tag{1.6}$$

On this basis, we may conclude that

$$\frac{P}{\pi}-\frac{1}{\pi}\int_{-1}^{1}\frac{f'(t)\sqrt{1-t^2}}{t-x}dt+\frac{1}{\pi^2}\int_{-1}^{1}\frac{\sqrt{1-t^2}}{t-x}dt\int_{-1}^{1}F_t'\left(\frac{t-y}{\lambda}\right)\frac{\omega(y)}{\sqrt{1-y^2}}dy\in C\,(-1,1 \tag{1.7}$$

if the above-mentioned properties of the functions $f(x)$ and $F(t)$ are taken into account, and it is also assumed that $\omega(t)\in C$ $(-1, 1)$. Hence, it remains to be shown that

$$\int_{-1}^{1}\frac{\sqrt{1-t^2}}{t-x}dt\int_{-1}^{1}\operatorname{sgn}(t-y)\frac{\omega(y)}{\sqrt{1-y^2}}dy\in C\,(-1,1) \tag{1.8}$$

if $\omega(y)\in C$ $(-1, 1)$. To do this, let us rewrite the inner integral in (1.8) as

$$N(t)=2\int_{-1}^{t}\frac{\omega(y)}{\sqrt{1-y^2}}dy-\int_{-1}^{1}\frac{\omega(y)}{\sqrt{1-y^2}}dy \tag{1.9}$$

Evidently

$$N'(t)=2\omega(t)\,(1-t^2)^{-1/2}$$

which means that

$$N(t)\in H_0^{\alpha}(-1,1), \alpha=1/2$$

Hence, the validity of the condition (1.8) follows on the basis of (1.6). It has therefore been proved that $\omega(x)\in C$ $(-1, 1)$.

Let us turn to the construction of an asymptotic solution of the integral Eq. (1.1) for large λ, or equivalently, of (1.3). Let us expand the functions $F_i(t)$ $(i = 1, 2, 3)$, in (1.2), in power series in the neighborhood of $t = 0$ thus:

$$F_i(t)=a_{i1}t^2+a_{i2}t^4+a_{i3}t^6+\ldots \tag{1.10}$$

Let the radii of convergence of these series be ρ_i, respectively. Then all that follows, based on (1.10), will have meaning at least for

$$\lambda>2/\operatorname{Inf}\rho_i \tag{1.11}$$

Let us substitute $F_i(t)$ in the form (1.10) into (1.2), and then (1.2) into (1.3). Let us seek the solution of (1.3) as

$$\omega(x)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\omega_{mn}(x)\lambda^{-m}\ln^n\lambda \tag{1.12}$$

Substituting (1.12) into (1.3) and equating terms on the right- and left-hand sides, which are of identical powers in λ^{-1} and $\ln\lambda$, we obtain expressions for $\omega_{mn}(x)$ for all $n > [m/2]$ (here $[y]$ is the integer part of y) as follows:

$$\omega_{00}(x) = \frac{P}{\pi} - \frac{1}{\pi}\int_{-1}^{1} \frac{f'(t)\sqrt{1-t^2}}{t-x}\,dt \tag{1.13}$$

$$\omega_{10}(x) = \frac{a_{20}}{\pi^2}\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x}\,dt \int_{-1}^{1} \frac{\omega_{00}(\tau)\,\mathrm{sgn}\,(t-\tau)}{\sqrt{1-\tau^2}}\,d\tau$$

$$\omega_{20}(x) = \frac{1}{\pi^2}\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x}\,dt \int_{-1}^{1} [(t-\tau)(2a_{11}\ln|t-\tau| + 2a_{31} + a_{11})\,\omega_{00}(\tau) + a_{20}\,\mathrm{sgn}\,(t-\tau)\,\omega_{10}(\tau)]\frac{d\tau}{\sqrt{1-\tau^2}}$$

$$\omega_{21}(x) = -\frac{2a_{11}}{\pi^2}\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x}\,dt \int_{-1}^{1} \frac{(t-\tau)\,\omega_{00}(\tau)}{\sqrt{1-\tau^2}}\,d\tau$$

$$\omega_{30} = \frac{1}{\pi^2}\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x}\,dt \int_{-1}^{1} \{[2a_{11}(t-\tau)\ln|t-\tau| + (a_{11}+2a_{31})(t-\tau)]\,\omega_{10}(\tau) + a_{20}\,\mathrm{sgn}\,(t-\tau)\,\omega_{20}(\tau) + 3a_{21}(t-\tau)^2\,\mathrm{sgn}\,(t-\tau)\,\omega_{00}(\tau)\}\frac{d\tau}{\sqrt{1-\tau^2}}$$

$$\omega_{31}(x) = \frac{1}{\pi^2}\int_{-1}^{1} \frac{\sqrt{1-t^2}}{t-x}\,dt \int_{-1}^{1} [-2a_{11}(t-\tau)\,\omega_{10}(\tau) + a_{20}\,\mathrm{sgn}\,(t-\tau)\,\omega_{21}(\tau)]\frac{d\tau}{\sqrt{1-\tau^2}} \qquad \text{etc.}$$

Let us limit ourselves to the consideration of the important particular case $f(x) = \delta = \text{const}$. Successively evaluating the quadratures in (1.13), we will have

$$\omega_{00}(x) = \pi^{-1}P, \qquad \omega_{10}(x) = 4\pi^{-3}a_{20}PS_1(x) \tag{1.14}$$

$$\omega_{20}(x) = \pi^{-1}P\,[(a_{11}(3/2 - \ln 2) + a_{31})(1-2x^2) + 32\pi^{-4}a_{20}^2(S_2(x) - D)$$

$$\omega_{21}(x) = -\pi^{-1}Pa_{11}(1-2x^2), \qquad \omega_{31}(x) = -2\pi^{-3}a_{11}a_{20}S_4(x)$$

$$\omega_{30}(x) = P\pi^{-3}\{8/9\,a_{11}a_{20}S_3(x) + [6a_{21}(1+2x^2) - 128\pi^{-4}a_{20}^3D]\,S_1(x) + [9a_{21} + 2(a_{11}(3/2 - \ln 2) + a_{31})a_{20}]\,S_4(x) + 8/3\,a_{21} + 64\pi^{-4}a_{20}^3S_5(x)\}$$

Here (*)

$$S_1(x) = (1-2x^2) + 2\sqrt{1-x^2}\sum_{k=1}^{\infty}\frac{\sin[(2k+1)\arccos x]}{(2k+1)^2} \tag{1.15}$$

$$S_2(x) = (1-x^2)\sum_{k=1}^{\infty}\frac{U_{2k}(x)}{(4k^2-1)^2}, \qquad D = \sum_{k=1}^{\infty}\frac{4k}{(4k^2-1)^3} = 0.1508$$

$$S_3(x) = -(1-2x^2) + 144\sqrt{1-x^2}\sum_{k=0}^{\infty}\frac{\sin[(2k+3)\arccos x]}{(2k+1)^2(2k+3)^2(2k+5)^2}$$

$$S_4(x) = \frac{1}{3} + (1-2x^2) + x(1-x^2)\ln\frac{1-x}{1+x}, \qquad S_5(x) = \int_{-1}^{1}\frac{\sqrt{1-t^2}}{t-x}\,dt\int_{0}^{t}\frac{S_2(\tau)\,d\tau}{\sqrt{1-\tau^2}},$$

$$U_{2k+2}(x) = -2(1-2x^2)\,U_{2k}(x) + 16k/(4k^2-1) - U_{2k-2}(x)$$

$$U_0(x) \equiv 0, \qquad U_2(x) = 4 + 2x\ln\frac{1-x}{1+x}$$

*) The series in the expression for $S_1(x)$ are tabulated in [2].

Therefore, an asymptotic solution of the form (1.12) to $O(\lambda^{-4})$ accuracy has been obtained for the case $f(x) = \delta$.

The series in (1.15) may be tabulated with respect to x once and forever. As computations with an error not exceeding 0.7% have shown, the function $S_2(x)$ may be replaced for all $x \in [-1, 1]$ by Expression:

$$S_2^*(x) = \left(0.4356 + 0.1321\, x^2 + 0.2494\, x \ln \frac{1-x}{1+x}\right)(1-x^2) \tag{1.16}$$

It should be noted that by thus approximating $S_2(x)$ we do not alter the character of its structure because, as is easily seen, the function $S_2(x)$ has the form

$$\left[f_1(x) + f_2(x) \ln \frac{1-x}{1+x}\right](1-x^2)$$

Here $f_1(x)$ and $f_2(x)$ are continuous functions for $x \in [-1, 1]$. Using (1.16), we obtain the following approximate expression for $S_5(x)$:

$$\begin{aligned} S_5^*(x) = 0.3547 - 0.8463\, x^2\, 10 + 0.3442\, x^4 + x(1-x^2) \ln \frac{1-x}{1+x} \times \\ \times (0.1180 + 0.03305\, x^2) - 0.04156\, (1-x^2)^2 \ln^2 \frac{1-x}{1+x}\, 10 + 0.3026 S_1(x) \end{aligned} \tag{1.17}$$

Finally, utilizing (1.4) and (1.14) to (1.17), we obtain for P for the case $f(x) = \delta$:

$$\begin{aligned} \pi\delta P^{-1} = \ln 2\lambda\, [1 - a_{11}\lambda^{-2} + 0.1801\, a_{11}a_{20}\lambda^{-3} + O(\lambda^{-4})] + a_{30} + 0.8106 a_{20}\lambda^{-1} + \\ + (a_{31} + a_{11} - 0.03287\ \ a_{20}^2)\lambda^{-2} + \\ + (1.442\ \ a_{21} - 0{,}2702\ a_{11}a_{20} - 0.1807\ \ a_{31}a_{20} - 0.02450\, a_{20}^3)\, \lambda^{-3} + O(\lambda^{-4}) \end{aligned} \tag{1.18}$$

2. Axisymmetric contact problems for an infinite circular elastic cylinder and an elastic space with an infinite circular cylindrical cavity. Let us consider the problem of interaction between a stiff belt and the surface of a cylinder, and between a stiff bushing and the surface of a cavity. Let us assume frictional forces are absent in the contact domain, and a load absent outside the contact domain.

By operational calculus methods the problems mentioned may be reduced to the determination of the contact pressures $q(z)$ from the integral equation [3 to 5]:

$$\int_{-a}^{a} q(\tau) K\left(\frac{\tau - z}{R}\right) d\tau = \pi\Delta\gamma \qquad \left(\begin{array}{l} |z| \leqslant a \\ \Delta = {}^1/_2\, E\,(1-\nu^2)^{-1} \end{array}\right) \tag{2.1}$$

Here a is half the belt or bushing width, R the radius of the cylinder or cavity, γ is the value of penetration of the belt or bushing into the surface of the cylinder or the cavity. The kernel $K(t)$ has the form

$$K(t) = \int_0^\infty \frac{L(u)}{u} \cos ut\, du \qquad \left(t = \frac{\tau - z}{R}\right) \tag{2.2}$$

for the problem (a) of interaction between a belt and the surface of a cylinder

$$L(u) = [u^2(\Omega_1^2 - 1) - 2(1-\nu)]^{-1} u \qquad (\Omega_1(u) = I_0(u) \,/\, I_1(u)) \tag{2.3}$$

for the problem (b) of interaction between a bushing and a cavity surface

$$L(u) = [u^2(1 - \Omega_2^2) + 2(1-\nu)]^{-1}\, u \qquad (\Omega_2(u) = K_0(u) \,/\, K_1(u)) \tag{2.4}$$

Here $I_0(u)$, $I_1(u)$ and $K_0(u)$, $K_1(u)$ are the Weber and Macdonald functions.

It is easy to see that the functions $L(u)$ defined by (2.3) and (2.4) for large values, may also be represented by the following asymptotic expansions:

$$L(u) = 1 + c_1 u^{-1} + c_2 u^{-2} + c_3 u^{-3} + O(u^{-4}) \tag{2.5}$$

Let us present the values of the constants c_i for $\varkappa = 0.3$

For problem (a)

$$c_1 = 0.4000, \quad c_2 = -1.285, \quad c_3 = -1.452$$

For problem (b)

$$c_1 = -0{,}4000 \quad c_2 = -0{,}965 \quad c_3 = 1.986$$

Now if the integrals

$$-\ln|t| = \int_0^\infty \frac{\cos ut - e^{-u}}{u}\,du, \qquad \frac{\pi}{2}\operatorname{sgn} t = \int_0^\infty \frac{\sin ut}{u}\,du \tag{2.6}$$

are used, then the kernel $K(t)$ may be represented as:

$$K(t) = -\ln|t| - {}^1/_2\,\pi c_1|t| + {}^1/_2\,c_2t^2\ln|t| - {}^3/_4\,c_2t^2 + {}^1/_{12}\,\pi c_3|t|^3 +$$
$$+\int_0^\infty \Big\{[u^3L(u) - u^3 - c_1u^2 - c_2u - c_3]\cos ut + u^3e^{-u} + c_1u^2 - \tag{2.7}$$
$$- c_2u\left(\frac{1}{2}u^2t^2e^{-u} - 1\right) - c_3\left(\frac{1}{2}u^2t^2 - 1\right)\Big\}\frac{du}{u^4}$$

We hence obtain the following asymptotic representation for the kernel $K(t)$ for small t (or equivalently, for large $\lambda = R/a$):

$$K(t) = -\ln|t| + a_{20}|t| + a_{30} + a_{11}t^2\ln|t| + a_{31}t^2 + a_{21}|t|^3 \tag{2.8}$$

Here

$$a_{20} = -\frac{\pi}{2}c_1, \quad a_{11} = \frac{c_2}{2}, \quad a_{21} = \frac{\pi}{12}c_3, \quad a_{30} = \int_0^\infty \frac{L(u) - 1 + e^{-u}}{u}\,du$$

$$a_{31} = -\frac{3}{4}c_2 + \frac{1}{2}\int_0^\infty [u^2 - u^2L(u) + c_1u + c_2(1 - e^{-u})]\,\frac{du}{u} \tag{2.9}$$

Calculations yield

For problem (a) ($\nu = 0.3$)

$a_{20} = -0.628, \quad a_{11} = -0.642, \quad a_{21} = -0.380, \quad a_{30} = -0.552, \quad a_{31} = 1.501$

For problem (b)

$a_{20} = 0.628, \quad a_{11} = -0.482, \quad a_{21} = 0.520, \quad a_{30} = -0.459, \quad a_{31} = -0.336$

Let us interchange the variables in (2.1), and let us introduce the notation:

$$\tau = at, \qquad z = ax, \qquad q(at) = \varphi(t), \qquad \Delta\gamma / a = \delta \tag{2.10}$$

Then taking (2.8) into account, (2.1) takes the form (1.1), (1.2) and (1.10).

Thus it follows that the asymptotic solution, for large λ, of the considered contact problems is given by (1.14) to (1.18). It should still be noted that

$$Q = \int_{-a}^{a} q(t)\,dt = aP \tag{2.11}$$

To verify the results obtained herein, and to elucidate the limits of their application, approximate solutions of the considered contact problems were obtained for $\nu = 0.3$ by a method expounded in [4]:

For problem (a)

$$\begin{aligned} (\lambda = 2) \quad & \omega(x) = \delta\,(1.229 - 0.421x^2 + 0.0901\,x^4) \\ (\lambda = 4) \quad & \omega(x) = \delta\,(0.688 - 0.0847\,x^2 + 0.0243\,x^4) \end{aligned} \tag{2.12}$$

For problem (b)

$$\begin{aligned} (\lambda = 2) \quad & \omega(x) = \delta\,(0.931 - 0{,}182\,x^2 - 0.0517\,x^4) \\ (\lambda = 4) \quad & \omega(x) = \delta\,(0.605 - 0{,}0634x^2 - 0.0213\,x^4) \end{aligned} \tag{2.13}$$

Given for comparison in Table 1 are some results of computations utilizing (1.14) to (1.18) and (2.13), (2. 12). Judging by the data presented, the approximate solution obtained herein

$$\omega(x) = \sum_{i=0}^{3}\sum_{j=0}^{[i/2]} \omega_{ij}(x)\,\lambda^{-i}\ln^j\lambda + O(\lambda^{-4}) \tag{2.14}$$

yields good results for the considered contact problems for all values of the parameter $\lambda \in [2, \infty]$. The greatest discrepancy, reaching 5%, is observed for $\lambda = 2$.

Table 1

		$\omega(0)\delta^{-1}$		$\omega(0,5)\delta^{-1}$		$\omega(1)\delta^{-1}$		$P(\pi\delta)^{-1}$	
	λ	2	4	2	4	2	4	2	4
a	(1.14)—(1.18)	1.075	0.686	1.075	0.667	0.923	0.627	1.038	0.653
	(2.12)	1.229	0.688	1.129	0.668	0.898	0.628	1.053	0.655
b	(1.14)—(1.18)	0.988	0.603	0.888	0.586	0.666	0.520	0.803	0.565
	(2.13)	0.931	0.605	0.882	0.588	0.697	0.520	0.821	0.565
c	(3.3)	1.026	0.642	0.961	0.624	0.762	0.571	0.894	0.606
	(3.4)	1.062	0.643	0.990	0.625	0.786	0.570	0.922	0.607

3. Axisymmetric problem of interaction between an elastic belt and an infinite elastic circular cylinder. Let us consider the problem of interaction between an elastic belt of radius R^* and the surface of an elastic cylinder of radius $R = R^* + \varepsilon(\varepsilon / R \ll 1)$. Let the elastic constants of the belt and the cylinder be E, ν and E^*, ν^*, respectively. We assume that there are no friction forces between the surfaces of the belt and the cylinder, and the cylinder surface is not loaded outside the belt. The condition for contact between the belt and cylinder may evidently be written as

$$u(R, z) - u(R^*, z) = -\varepsilon, \qquad |z| \leqslant a \tag{3.1}$$

where $u(R, z)$ is the radial displacement of points of the cylinder surface, $u(R^*, z)$ the radial displacement of point of the belt surface, and a is half the belt width. In the contact domain $|z| \leqslant a$ an unknown contact pressure $q(z)$ acts. Let us formulate the problem of determining this pressure.

Now, as is done in the well-known Hertz theory of contact between two elastic bodies, let us assume that the radial displacements of the belt surface due to the pressure $q(z)$ may be approximated with sufficient accuracy by radial displacements of the surface of an infinite cylindrical cavity of radius R in an elastic space due to the same pressure. Then, by utilizing (2.1) to (2.4), we easily form an integral equation to determine the contact pressure. Let us write this equation in the form (2.1) with a kernel such as (2.2), wherein

$$L(a) = \frac{\Delta a}{\Delta + \Delta^*}\left[u^2(\Omega_1^2 - 1) - 2(1-\nu)\right]^{-1} + \frac{\Delta u}{\Delta + \Delta^*}\left[u^2(1 - \Omega_2^2) + 2(1-\nu^*)\right]^{-1}$$

$$\Delta^* = {}^1/_2 E^*(1 - \nu^{*2})^{-1}, \qquad \gamma = \Delta^*(\Delta + \Delta^*)^{-1}\varepsilon \tag{3.2}$$

For small t the kernel $K(t)$ of the problem under consideration may be represented in the form (2.8), just as is done in Section 2. If we, henceforth, put $\nu = \nu^* = 0.3$ the coefficients of the asymptotic expansion (2.8) will then be

$$a_{20} = -0.628\mu^* + 0.628\mu, \qquad a_{11} = -0.642\mu^* - 0.482\mu$$

$$a_{21} = -0.380\mu^* + 0.520\mu, \qquad a_{30} = -0.552\mu^* - 0.459\mu$$

$$a_{31} = 1.501\mu^* - 0.336\mu, \quad \mu^* = \Delta^*(\Delta + \Delta^*)^{-1}, \quad \mu = \Delta(\Delta + \Delta^*)^{-1}$$

For known coefficients a_{ij} the solution of the problem will be determined, as before, by (1.14) to (1.18). These formulas simplify significantly but just for one particular, practically important, case. Namely, let us put $\Delta^* = \Delta$ then

$$\mu^* = \mu = {}^1/_2 \text{ and } a_{20} = 0, \; a_{11} = -0.562, \; a_{21} = 0.070, \; a_{30} = -0.506, \; a_{31} = 0.582$$

Taking account of the coefficient a_{20} vanishing, we obtain the solution in conformity with (1.14) to (1.18), as

$$\omega(x) = Q(\pi a)^{-1}\{1 + \lambda^{-2}({}^3/_2\, a_{11} + a_{31})(1 - 2x^2) - a_{11}\lambda^{-2}\ln 2\lambda\,(1 - 2x^2) +$$

$$+ \pi^{-2}\lambda^{-3}[6a_{21}(1 + 2x^2)S_1(x) + 9a_{21}S_4(x) + {}^8/_3\, a_{21} + O(\lambda^{-4}), \qquad Q = aP$$

$$\pi\delta P^{-1} = \ln 2\lambda\,[1 - a_{11}\lambda^{-2} + O(\lambda^{-4})] + a_{30} + (a_{31} + a_{11})\lambda^{-2} + 1{,}442a_{21}\,\lambda^{-3} + O(\lambda^{-4})$$

$$q(z) = \omega(z / a)\, a\,(a^2 - z^2)^{-1/2}, \quad \delta = \Delta\varepsilon / 2a \tag{3.3}$$

Approximate solutions of the problem obtained by the method of [4] for the previously assigned conditions $E = E^*$, $\nu = \nu^* = 0.3$ are

$$(\lambda = 2) \qquad \omega(x) = \delta\,(1.062 - 0.294\,x^2 + 0.0176\,x^4)$$

$$(\lambda = 4) \qquad \omega(x) = \delta\,(0.643 - 0.0712\,x^2 - 0.00160\,x^4) \tag{3.4}$$

Given for comparison in Table 1 are results of computations utilizing (3.3) and (3.4). It is seen from these data that, as in the previous problems, (3.3) may reliably be utilized in engineering analyses if $\lambda \geqslant 2$.

For convenience of practical utilization of the results obtained herein, values of the functions $S_i(x)$ ($i = 1, 2, 3, 4, 5$), defined by (1.15), are given in Table 2.

Table 2

x	$S_1(x)$	$S^*_2(x)$	$S_3(x)$	$S_4(x)$	$S^*_5(x)$
0	0.8320	0.4356	−1.628	1.333	0.6065
0.1	0.8178	0.4276	−1.578	1.293	0.5898
0.2	0.7750	0.4039	−1.429	1.175	0.5403
0.3	0.7029	0.3650	−1.192	0.9843	0.4604
0.4	0.6003	0.3127	−0.8818	0.7286	0.3536
0.5	0.4650	0.2487	−0.5087	0.4211	0.2260
0.6	0.2938	0.1765	−0.1098	$0.8110\cdot10^{-1}$	$-0.8170\cdot10^{-1}$
0.7	$0.8156\cdot10^{-1}$	0.1007	+0.2593	+0.2661	$-0.6830\cdot10^{-1}$
0.8	−0.1805	$0.2945\cdot10^{-1}$	+0.6380	−0.5794	−0.2146
0.9	−0.5117	$-0.2245\cdot10^{-1}$	+0.8267	−0.7903	−0.3458
1.0	−1.000	0.000	1.000	−0.6667	−0.4500

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